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The photon: a collective excitation of fermion fields

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Abstract. The photon is described as a collective excitation of a fermion field. The basic short range self-interactions of this field are assumed to be gauge invariant and nonlocal. By breaking this symmetry, a one-fermion Green function of the Dirac type can be found. Consequently, a collective excitation of zero mass, that is, the photon, appears as a pole of the two-fermion Green function. Thus the usual electron and photon field theory can be qualitatively reconstructed.

1. Introduction

Since the photon has spin one and no mass, it is tempting to consider it as a bound state of two fermions. The idea is old and was discussed by many physicists: Ascoli and Heisenberg (1957), Bialynicki-Birula (1963), Bjorken (1963), de Broglie (1943), Budini and Furlan (1972), Heisenberg (1934, 1957, 1966), Jouvet (1957) and Nambu and Jona-Lasinio (1961).

In general, the starting point of such a discussion is a nonlinear Lagrangian of the form:

$$\mathscr{L}(x) = \bar{\psi}(x)(i\gamma^{\nu}\partial_{\nu})\psi(x) + \sum_{j} \lambda_j \bar{\psi}(x)\mathscr{O}_j\psi(x)\bar{\psi}(x)\mathscr{O}_j\psi(x), \tag{1.1}$$

where the operators $\mathscr{O}_j$ are products of operators γ_μ

$$\mathscr{O}_j \in \{1, \gamma_5, \gamma_\nu, \gamma_5\gamma_\nu, \gamma_\mu\gamma_\nu\}. \tag{1.2}$$

However, serious difficulties appear. The perturbation diagrams are very divergent and the introduction of bound states in a relativistic field theory is also troublesome, see Cutkovsky (1964), Itzykson *et al* (1970), Wick (1954). Moreover, there is no decisive reason for replacing the mass term of the Dirac Lagrangian, and only this term, by a biquadratic interaction.

In this article, the question is re-examined and we show how the photon may appear in a natural way as a collective excitation of a self-interacting fermion field.

2. A gauge invariant model

The fact that the photon has no mass seems to indicate that this particle is a Goldstone particle corresponding to a broken symmetry. The nature of this symmetry should be very obvious. We think that this important broken symmetry is gauge invariance, though, of course, isopin conservation is also broken in the process.

The Dirac equation in the absence of coupling with an electromagnetic field, is not gauge invariant in itself. It is not preserved by the transformation $\psi(x) \to \exp(i\chi(x))\psi(x)$. Gauge invariance is a property of the whole theory but the existence of the electromagnetic field is necessary for insuring this property. In discussions of spontaneous broken symmetries contained in the literature, the broken symmetry is usually taken to be independent of space. Here the gauge symmetry is broken at each point in space–time. This difference must probably be related to the fact that the photon can be described by a field of arbitrary gauge: the gauge invariance properties of the collective excitation must reflect the gauge symmetry of the whole theory, at each point of space–time.

In the following, for the sake of simplicity, we consider the photon as a collective excitation of the electron–positron field only. We assume the existence of an action W which is a functional of the electron–positron field. As this action has to be gauge invariant, it must be a function of terms of the form $\psi(x)\mathcal{O}\psi(x)$ where $\mathcal{O}$ is a product of matrices γ_ν (it does not contain any derivative).

Thus, retaining only the terms of lowest order, we may write:

$$W(\psi, \bar{\psi}) = -m_0 \int \bar{\psi}(x)\psi(x)\, \mathrm{d}^4x + \sum_j \int \bar{\psi}(x)\mathcal{O}_j\psi(x)F_j(x-y)\bar{\psi}(y)\mathcal{O}_j\psi(y)\, \mathrm{d}^4x\, \mathrm{d}^4y.$$

The functions $F_j(x-y)$ describe very short range interactions and their properties are such that W is a relativistic invariant.

The field self-interaction is nonlocal. This is an unconventional situation and a source of difficulties. There is no Lagrangian and the usual formalism cannot be directly applied; we do not even know how to proceed and in particular, how to quantize the fields.

On the other hand, the nonlocal character of the interaction is inevitable; otherwise, the fields would not propagate (strictly speaking a local term does not contain any derivative of the fields) and strong ultraviolet divergences would appear.

Fortunately, systems of particles with nonlocal interactions are not untractable. The usual field theory can be generalized; for instance, we may use a method introduced by Bell and Skyrme which will be described in the next section. The method may not be completely realistic and correct but, at least, it has the merit of leading to simple and well defined prescriptions. It amounts to a straightforward generalization of Wick's theorem. The same result could be found by the function integral method, properly defined by Berezin (1966) in his book on second quantization.

3. Field theory—the Bell–Skyrme method

The Bell–Skyrme method (Bell 1962, Skyrme 1955) enables us to express directly the many-body Green functions $G(x_1, \ldots, x_n | y_1, \ldots, y_n)$ in terms of the action $W(\psi, \bar{\psi})$.

We start by assuming that the fields ψ and $\bar{\psi}$ which appear in $W(\psi, \bar{\psi})$, anticommute:

$$[\psi(x), \bar{\psi}(y)]_+ = [\psi(x), \psi(y)]_+ = [\bar{\psi}(x), \bar{\psi}(y)]_+ = 0. \tag{3.1}$$

By definition, they are given by the sum of two fields

$$\begin{aligned} \psi(x) &= a(x) + \bar{b}(x) \\ \bar{\psi}(x) &= \bar{a}(x) - b(x). \end{aligned} \tag{3.2}$$

These fields a and b obey the following anticommutation relations:

$$[a(x), \bar{a}(y)]_+ = [b(x), \bar{b}(y)]_+ = \beta^{-1}\delta(x-y)I,$$

(here β is an arbitrary constant and I the unit matrix) and all the other anticommutators vanish.

The state $|S\rangle$ is the Skyrme vacuum:

$$a(x)|S\rangle = 0 \qquad b(x)|S\rangle = 0. \tag{3.3}$$

We set

$$A(\psi, \bar{\psi}) = W(\psi, \bar{\psi}) - \mathrm{i}(\beta - 0)\int \mathrm{d}^4 x \bar{\psi}(x)\psi(x) \tag{3.4}$$

and introduce the 'mean value' of any functional $B(\psi, \bar{\psi})$ of the fields by

$$\langle\langle B(\psi, \bar{\psi})\rangle\rangle \equiv \frac{\langle S|\exp(\mathrm{i}A(\psi, \bar{\psi}))B(\psi, \bar{\psi})|S\rangle}{\langle S|\exp(\mathrm{i}A(\psi, \bar{\psi}))|S\rangle}. \tag{3.5}$$

Finally, the Green functions are defined by

$$G(x_1, \ldots, x_n|y_1, \ldots, y_n) = \langle\langle\psi(x_1)\ldots\psi(x_n)\bar{\psi}(y_1)\ldots\bar{\psi}(y_n)\rangle\rangle. \tag{3.6}$$

In order to avoid divergent terms, we must consider any product of the form $\psi(x)\mathcal{O}\psi(x)$ appearing in $W(\psi, \bar{\psi})$ as a normal product (its mean value in Skyrme's vacuum vanishes) and we write:

$$:\bar{\psi}(x)\mathcal{O}\psi(x): = \bar{a}(x)\mathcal{O}[a(x) + \bar{b}(x)] + [a(x) + \bar{b}(x)]\tilde{\mathcal{O}}b(x). \tag{3.7}$$

As, in ordinary field theory, $G(x_1, \ldots, x_n|y_1, \ldots, y_n)$ can be represented by diagrams as follows.

If $W(\psi, \bar{\psi})$ is replaced by

$$W_0(\psi, \bar{\psi}) = \bar{\psi}Z\psi, \tag{3.8}$$

where Z is an arbitrary operator (eg $(\gamma \cdot p - m)$), the corresponding one-body Green operator obtained by direct application of equations (3.5) and (3.6) is (see appendix 1):

$$G = \frac{1}{Z + \mathrm{i}0}. \tag{3.9}$$

In this case, we remark that the many-body Green function (equation 3.6) can be calculated by direct application of Wick's theorem (Wick 1950, Gaudin 1960, Balian and Brezin 1969) to the mean value of the product $\psi(x_1)\ldots\psi(x_n)\bar{\psi}(y_1)\ldots\bar{\psi}(y_n)$.

In the same way, in the general case, we may write

$$W(\psi, \bar{\psi}) = W_0(\psi, \bar{\psi}) + [W(\psi, \bar{\psi}) - W_0(\psi, \bar{\psi})] \tag{3.10}$$

and expand $\exp(\mathrm{i}A(\bar{\psi}, \psi))$ with respect to $[W(\psi, \bar{\psi}) - W_0(\psi, \bar{\psi})]$. Each term can be calculated by application of Wick's theorem and each contraction of ψ and $\bar{\psi}$ introduces a factor $(Z + \mathrm{i}0)^{-1}$. Thus we find a complete diagram representation of the Green function.

We see immediately that, if W is the integral of a local Lagrangian $\mathscr{L}(x)$, the preceding definitions coincide exactly with the usual ones, that is, the Green functions are expectation values in vacuum of the time ordered operators quantized according to the classical rules.

However, the present definition applies equally well when the interactions are nonlocal, and it leads to a straightforward generalization of the usual field theory diagrams. We have to show now that it is equivalent to the usual electron and photon field theory.

4. The self-consistent propagator Dirac equation

The electron propagator must be determined in a self-consistent way. Since the action $W_0(\psi, \bar{\psi})$ should be a relativistic invariant, we may write

$$W_0(\psi, \bar{\psi}) = \int \mathrm{d}^4 p \bar{\psi}(\boldsymbol{p}) u(\boldsymbol{p}) \psi(\boldsymbol{p}), \tag{4.1}$$

where $\bar{\psi}(\boldsymbol{p})$ and $\psi(\boldsymbol{p})$ are the Fourier transforms of $\psi(\boldsymbol{x})$ and $\bar{\psi}(\boldsymbol{x})$ and $u(\boldsymbol{p})$ a 4×4 matrix. The corresponding Green function is

$$G^0(p) = \frac{1}{u(\boldsymbol{p}) + \mathrm{i}0}. \tag{4.2}$$

Here, a diagram will be considered as irreducible if it cannot be separated into disconnected parts by cutting *one* or *two* electron lines.

We denote by $\Sigma(\boldsymbol{p}, \{u\})$ the irreducible self-energy. It is a functional of the interactions $F_j(\boldsymbol{x}-\boldsymbol{y})$ and of $u(\boldsymbol{p})$ (but it does not depend on m_0).

The Green function $G_0(\boldsymbol{p})$ which corresponds to the ordinary unrenormalized electron propagator must be determined by the self-consistent equation:

$$\Sigma(\boldsymbol{p}, \{u\}) - u(\boldsymbol{p}) + m_0 = 0. \tag{4.3}$$

The original gauge symmetry is broken, if this equation has solutions $u(\boldsymbol{p})$ which are not constants.

In order to establish a correspondence with the Dirac equation we must look for a solution of the form

$$u(\boldsymbol{p}) = C(p^2)(\gamma \cdot \boldsymbol{p} - M(p^2)) \tag{4.4}$$

(the metric is $1, -1-1, -1$) where $C(p^2)$ and $M(p^2)$ are functions determined in a self-consistent way.

A simple approximation consists in retaining in $\Sigma(\boldsymbol{p}, \{u\})$ the first order terms (generalized Hartree–Fock approximation: see figure 3). In this case, simple examples can be given where explicit operators $u(\boldsymbol{p})$ of the form (4.4) can be found.

This shows, at least, that it is not unrealistic to assume that equation (4.3) has solutions of type (4.4).

The unrenormalized electron mass associated with $u(\boldsymbol{p})$ is a solution of the equation:

$$M(m^2) = m \tag{4.5}$$

and if

$$mM'(m^2) \ll 1, \tag{4.6}$$

we may replace $G^0(p)$ a Dirac propagator:

$$G^0(p) \simeq \frac{1}{C(p^2)(\gamma p - m)}, \tag{4.7}$$

where $C(p^2)$ is a simple renormalization factor.

5. The electron–positron two-body Green function

The photon will be identified with a bound state of the electron–positron system. This state appears as a pole in the electron–positron Green function. Let us denote by $K(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$ the irreducible four point kernel (see figure 1).

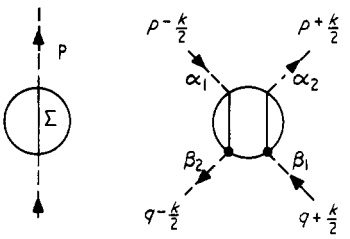

Figure 1. Irreducible self-energy $\Sigma(\boldsymbol{p}) \equiv \Sigma(\boldsymbol{p}, \{u\})$ and irreducible kernel $K(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$.

The T matrix $T(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$ associated with electron–positron scattering is a solution of the Bethe–Salpeter equation (see figure 2):

$$T_{\alpha_1\alpha_2\beta_2\beta_1}(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k}) = K_{\alpha_1\alpha_2\beta_2\beta_1}(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k}) + \int \mathrm{d}^4 l K_{\alpha_1\alpha_2\gamma_2\gamma_1}(\boldsymbol{p}, \boldsymbol{l}; \boldsymbol{k}) G^0_{\gamma_1\delta_1}(\boldsymbol{l}+\tfrac{1}{2}\boldsymbol{k}) G^0_{\delta_2\gamma_2}(\boldsymbol{l}-\tfrac{1}{2}\boldsymbol{k}) \times T_{\delta_1\delta_2\beta_1\beta_2}(\boldsymbol{l}, \boldsymbol{q}; \boldsymbol{k}). \tag{5.1}$$

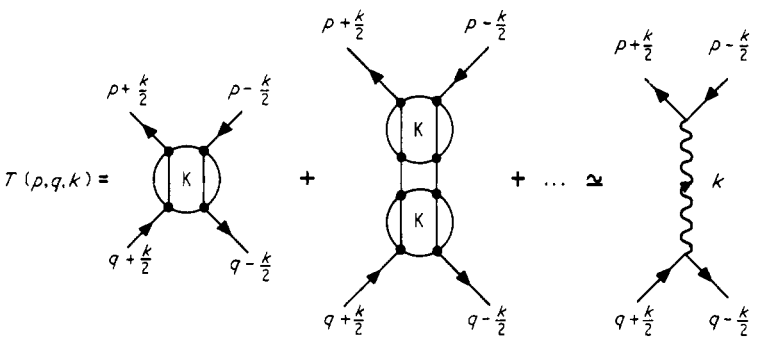

Figure 2. The sum of diagrams on the left represents the matrix $T(p, q; k)$; the wavy line represents the photon propagator.

We want to show that $T(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$ has a pole for $k^2 = 0$ and that near this pole, this function can be approximated by an expression of the form:

$$T_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k}) \simeq -\mathrm{i}\frac{e^2}{(2\pi)^4}\frac{1}{k^2+\mathrm{i}0}(\gamma^\nu)_{\alpha_1\beta_1}(\gamma_\nu)_{\alpha_2\beta_2}. \tag{5.2}$$

Here e is the unrenormalized electron charge; the experimental value e_R of the renormalized charge is $e_R = (4\pi/137)^{1/2}$.

The poles of $T(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$ and the residues of these poles can be found by solving the homogeneous equation:

$$\{1+\lambda(k^2)\}\mathcal{O}_{\alpha_1\alpha_2}(p;k) = \int \mathrm{d}^4 q K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})\{G^0(\boldsymbol{q}+\tfrac{1}{2}\boldsymbol{k})\mathcal{O}(q;k)G^0(\boldsymbol{q}-\tfrac{1}{2}\boldsymbol{k})\}_{\beta_2\beta_1}, \tag{5.3}$$

where $\lambda(k)$ is an eigenvalue.

For each value of $\boldsymbol{k}$, there exists a set of eigenfunctions $\mathcal{O}^{(n)}_{\alpha_1\alpha_2}(p, k)$ which are orthogonal in the following sense:

$$\int \mathrm{d}^4 p \,\mathrm{Tr}\, \mathcal{O}^{(n)}(\boldsymbol{p}; -\boldsymbol{k})G^0(\boldsymbol{p}+\tfrac{1}{2}\boldsymbol{k})\mathcal{O}^{(m)}(\boldsymbol{p}; \boldsymbol{k})G^0(\boldsymbol{p}-\tfrac{1}{2}\boldsymbol{k}) = A_n(\boldsymbol{k})\delta_{nm}, \tag{5.4}$$

a result which can be easily derived by using the symmetry property (see figure 1)

$$K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k}) = K_{\beta_1\beta_2\alpha_1\alpha_2}(\boldsymbol{p}, \boldsymbol{q}; -\boldsymbol{k}). \tag{5.5}$$

The existence of a solution $\lambda(k^2) = 0$ for $\boldsymbol{k} = 0$, is a consequence of the following Ward identity:

$$\frac{\hat{c}}{\hat{c}p^\mu}\Sigma(\boldsymbol{p}) = \int \mathrm{d}^4 q K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}; 0)\frac{\hat{\partial}}{\partial q^\mu}G^0_{\beta_2\beta_1}(q) \tag{5.6}$$

(here $\Sigma(\boldsymbol{p}) = \Sigma(\boldsymbol{p}, \{u\})$).

This identity can be obtained by differentiating term by term the contributions of the diagrams of $\Sigma(\boldsymbol{p})$. We see that by adding a momentum $\Delta\boldsymbol{p}$ to each internal line of a diagram of $\Sigma(\boldsymbol{p})$, we obtain a diagram of $(\boldsymbol{p}+\Delta\boldsymbol{p})$ without changing the values of the basic interactions, and this remark leads immediately to equation (5.6).

Differentiating equation (4.3) and comparing with equation (5.6), we find:

$$[\hat{c}_\mu u(p)]_{\alpha_1\alpha_2} = \int \mathrm{d}^4 q K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}; 0)[G^0(q)\hat{c}_\mu u(q)G^0(q)]_{\beta_1\beta_2}. \tag{5.7}$$

Thus, for $\boldsymbol{k} = 0$ and $\lambda(0) = 0$, we find a solution:

$$\mathcal{O}^{(0)}_\mu(\boldsymbol{p}; 0) = \hat{c}_\mu u(\boldsymbol{p}). \tag{5.8}$$

Consequently, for $k^2 = 0$ and $\lambda(0) = 0$, a solution also exists. This fact can be formally established by perturbation theory. We write:

$$\mathcal{O}^{(0)}_\mu(p; \boldsymbol{k}) = \mathcal{O}^{(0,0)}_\mu(p) + \sum_{s=1}^{\infty} k^{\nu_1}\ldots k^{\nu_s}\mathcal{O}^{(0,s)}_{\mu,\nu_1,\ldots,\nu_s}(\boldsymbol{p}), \tag{5.9}$$

(the small parameter in this expansion is $\boldsymbol{k}\cdot\boldsymbol{p}$ and $\mathcal{O}^{(0,0)}_\mu(\boldsymbol{p}) \equiv \mathcal{O}^{(0)}_\mu(\boldsymbol{p}; 0) = \hat{c}_\mu u(\boldsymbol{p})$). Since $k^2 = 0$, the tensor $k^{\nu_1}\ldots k^{\nu_s}$ is traceless and irreducible. Therefore, inserting this expansion in equation (5.3) which can be written in a symbolic way:

$$\{1+\lambda(k^2)\}\mathcal{O}(\boldsymbol{p}; \boldsymbol{k}) = \int \mathrm{d}^4 q \Omega(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})\mathcal{O}(\boldsymbol{q}; \boldsymbol{k}), \tag{5.10}$$

we find ($\lambda(\boldsymbol{k}^2) = 0$)

$$\mathcal{O}^{(0,s)}_{\mu,\nu_1\ldots\nu_s}(\boldsymbol{p}) - \int \mathrm{d}^4 q \Omega(\boldsymbol{p}, \boldsymbol{q}; 0)\mathcal{O}^{(0,s)}_{\mu,\nu_1\ldots\nu_s}(q) = Q(\{\mathcal{O}^{(0,s')}\}, s' < s). \tag{5.11}$$

Thus, $\mathcal{O}^{(0,s)}_{\mu,\nu_1,\ldots\nu_s}(\boldsymbol{p})$ is given by an integral equation in terms of functions of lower order $s'(s' < s)$. In general, since, by definition, this function is orthogonal to $\mathcal{O}^{(0)}(\boldsymbol{p}; \boldsymbol{k})$, the

equation has a solution and therefore all the coefficients $\mathcal{O}^{(0,s)}_{\nu_1,\ldots,\nu_s}(\boldsymbol{p})$ can be calculated step by step.

These remarks show that the T matrix given by equation (5.1) has a pole for $\boldsymbol{k} = 0$. Let us determine the residue of this pole.

Using equation (5.3) and the orthogonality relation (5.4), we find immediately that

$$K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p},\boldsymbol{q};\boldsymbol{k}) = \sum_n A_n^{-1}(\boldsymbol{k})(1+\lambda^{(n)}(\boldsymbol{k}))\mathcal{O}^{(n)}_{\alpha_1\alpha_2}(\boldsymbol{p};\boldsymbol{k})\mathcal{O}^{(n)}_{\beta_1\beta_2}(\boldsymbol{q};-\boldsymbol{k}). \tag{5.12}$$

In a similar way, equation (5.1) shows that the T matrix is given by:

$$T_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p},\boldsymbol{q};\boldsymbol{k}) = \sum_n (\lambda_n(k)A_n(k))^{-1}(1+\lambda^{(n)}(\boldsymbol{k}))\mathcal{O}^{(n)}_{\alpha_1\alpha_2}(\boldsymbol{p};\boldsymbol{k})\mathcal{O}^{(n)}_{\beta_1\beta_2}(\boldsymbol{q};-\boldsymbol{k}). \tag{5.13}$$

On the other hand, for small values of $\boldsymbol{k}$, equation (5.3) has a solution and

$$\lambda^{(0)}(\boldsymbol{k}) \simeq \lambda_0 k^2. \tag{5.14}$$

Thus, near the pole:

$$T_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p},\boldsymbol{q};\boldsymbol{k}) \simeq \frac{1}{k^2 A_0\lambda_0}[\partial_\mu u(\boldsymbol{p})]_{\alpha_1\alpha_2}[\partial^\mu u(\boldsymbol{q})]_{\beta_1\beta_2}, \tag{5.15}$$

where

$$A_0 = \tfrac{1}{4}\int \mathrm{d}^4p\,\mathrm{Tr}\{\partial^\mu u(\boldsymbol{p})(u(\boldsymbol{p})+\mathrm{i}0)^{-1}\partial_\mu u(\boldsymbol{p})(u(\boldsymbol{p})+\mathrm{i}0)^{-1}\}. \tag{5.16}$$

We must assume that, for relatively small values of p^2,

$$\frac{p^2C'(p^2)}{C(p^2)} \ll 1$$

$$\frac{p^2M'(p^2)}{M(p^2)} \ll 1. \tag{5.17}$$

These properties may well be related to the very short range character of the interaction $F_j(x)$.

Accordingly, we write:

$$T_{\alpha_1\alpha_2\beta_1\beta_2}(p,q,k) \simeq \frac{C(p^2)C(q^2)}{k^2A_0\lambda_0}[\gamma_\mu]_{\alpha_1\alpha_2}[\gamma^\mu]_{\beta_1\beta_2}. \tag{5.18}$$

The factor $C(p^2)C(q^2)$ must be eliminated by renormalizing the propagator $G^0(p)$ (see equation (4.7)).

Thus comparing equations (5.2) and (5.18), we find

$$\frac{e^2}{(2\pi)^4} = \frac{\mathrm{i}}{A_0\lambda_0}. \tag{5.19}$$

We remark that the photon propagator given by the T matrix in equation (5.18) is written in the Feynman gauge. This is somewhat surprising; other types of photon propagator corresponding to different gauges are possible and lead to the same physical S matrix values. Thus, *a priori*, some ambiguity could be expected, in the definition of the residue of the pole of the T matrix but it is not so. The T matrix is defined without ambiguity as a sum of well defined diagrams, in which appear only the one electron propagator $G_0(\boldsymbol{p})$ and basic interactions.

6. Crude calculation of the fine structure constant

An expression for λ_0 can be found by perturbation as will be shown now, in the case where $K(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k})$ is independent of $\boldsymbol{k}$ (as in the simple ladder approximation see figure 3):

$$K(\boldsymbol{p}, \boldsymbol{q}; \boldsymbol{k}) = K(\boldsymbol{p}, \boldsymbol{q}). \tag{6.1}$$

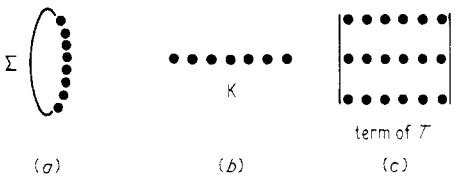

Figure 3. The diagrams corresponding to the linear approximation: Hartree–Fock approximation for Σ, ladder diagrams for T. The round dots represent the short range interactions $(F_j(\boldsymbol{x}-\boldsymbol{y}))$.

In § 5, we found that there exist solutions $\mathscr{C}_\mu^{(0)}(\boldsymbol{p}, \boldsymbol{k})$ for which $\lambda(k^2)$ vanishes when k^2 goes to zero. The scalar product $k^\mu \mathscr{C}_\mu^{(0)}(\boldsymbol{p}, \boldsymbol{k})$ is also a solution and may be expanded with respect to $\boldsymbol{k}$ (see equation (5.9)). In equation (5.3), let us introduce this expansion, namely:

$$\mathscr{C}^{(0)}(\boldsymbol{p}; \boldsymbol{k}) = k^\mu \mathscr{C}_\mu^{(0)}(\boldsymbol{p}, \boldsymbol{k}) = k\mathscr{C}_\mu^{(0)}(p) + k^\mu k^\nu \mathscr{C}_{\mu,\nu}^{(0,1)}(p) + \ldots. \tag{6.2}$$

The function $\mathscr{C}_{\mu,\nu}^{(0,1)}(p)$ is given by

$$\begin{aligned}
&\mathscr{C}_{\mu\nu}^{(0,1)}(\boldsymbol{p}) - \int \mathrm{d}^4 p K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}) [(u(\boldsymbol{q}) + \mathrm{i}0)^{-1} \mathscr{C}_{\mu\nu}^{(0,1)} (u(\boldsymbol{q}) + \mathrm{i}0)^{-1}]_{\beta_1\beta_2} \\
&\quad = \tfrac{1}{2} \int \mathrm{d}^4 q K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}) [(u(\boldsymbol{q}) + \mathrm{i}0)^{-1} \{\hat{c}_\nu u(\boldsymbol{q}) (u(\boldsymbol{q}) + \mathrm{i}0)^{-1} \hat{c}_\mu u(\boldsymbol{q}) \\
&\quad\quad - \hat{c}_\mu u(\boldsymbol{q}) (u(\boldsymbol{q}) + \mathrm{i}0)^{-1} \hat{c}_\mu u(\boldsymbol{q})\} (u(\boldsymbol{q}) + \mathrm{i}0)^{-1}]_{\beta_1\beta_2}.
\end{aligned} \tag{6.3}$$

From this equation, we deduce the result:

$$k^\mu k^\nu \mathscr{C}_{\mu\nu}^{(0,1)}(\boldsymbol{p}) = 0. \tag{6.4}$$

In equation (6.1), the second term disappears and this means that λ_0 can be obtained by first order calculation.

We find

$$\begin{aligned}
4\lambda_0 A_0 &\equiv \lambda_0 \operatorname{Tr} \int \mathrm{d}^4 p (k^\mu \mathscr{C}_\mu^{(0)}(\boldsymbol{p}; 0) G^0(\boldsymbol{p}) k_\nu \mathscr{C}_\nu^{(0)}(\boldsymbol{p}; 0) G^0(\boldsymbol{p})) \\
&= \tfrac{1}{24} (g^{\mu\nu} g^{\sigma\rho} + g^{\mu\sigma} g^{\nu\rho} + g^{\mu\rho} g^{\nu\sigma}) \left(\frac{\partial^2}{2\partial k^\sigma \partial k^\rho} \operatorname{Tr} \int \mathrm{d}^4 p \, \mathrm{d}^4 q [G^0(\boldsymbol{p}) \mathscr{C}_\mu^{(0)}(\boldsymbol{p}; 0) G^0(\boldsymbol{p})]_{\alpha_1\alpha_2} \right. \\
&\quad \left. \times K_{\alpha_1\alpha_2\beta_1\beta_2}(\boldsymbol{p}, \boldsymbol{q}) [G^0(\boldsymbol{q} + \tfrac{1}{2}\boldsymbol{k}) \mathscr{C}_\nu^{(0)}(\boldsymbol{q}, 0) G^0(\boldsymbol{q} - \tfrac{1}{2}\boldsymbol{k})]_{\beta_1\beta_2} \right)_{\boldsymbol{k}=0}
\end{aligned} \tag{6.5}$$

Then, after simplification, we obtain

$$\begin{aligned}
\lambda_0 A_0 = \tfrac{1}{96} (g^{\mu\nu} g^{\sigma\rho} + g^{\mu\sigma} g^{\nu\rho} + g^{\mu\rho} g^{\nu\sigma}) \operatorname{Tr} \int \mathrm{d}^4 p \hat{c}_\nu u(\boldsymbol{p}) \{ \tfrac{1}{2} \hat{c}_{\sigma\rho}^2 (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} \hat{c}_\nu u(\boldsymbol{p}) (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} \\
- \hat{c}_\sigma (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} \hat{c}_\nu u(\boldsymbol{p}) \hat{c}_\rho (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} + \tfrac{1}{2} (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} \hat{c}_\nu u(\boldsymbol{p}) \hat{c}_{\sigma\rho}^2 (u(\boldsymbol{p}) + \mathrm{i}0)^{-1} \}
\end{aligned} \tag{6.6}$$

and finally (with $u_\nu(\boldsymbol{p}) \equiv \partial_\nu u(\boldsymbol{p})$)

$$\lambda_0 A_0 = -\tfrac{1}{96}\,\mathrm{Tr}\int \mathrm{d}^4 p\,\partial_\nu\{u^\nu(\boldsymbol{p})(u(\boldsymbol{p})+\mathrm{i}0)^{-1}u_\mu(\boldsymbol{p})(u(\boldsymbol{p})+\mathrm{i}0)^{-1}u^\mu(\boldsymbol{p})(u(\boldsymbol{p})+\mathrm{i}0)^{-1}\}. \tag{6.7}$$

If, we take for $u(\boldsymbol{p})$ the classical value

$$u(\boldsymbol{p}) = C(\boldsymbol{\gamma}\,.\,\boldsymbol{p} - m) \tag{6.8}$$

after rotation of the integration contour in the integration contour in the p_0 plane, we find (see appendix 2):

$$\lambda_0 A_0 = \tfrac{1}{6}\mathrm{i}\pi^2. \tag{6.9}$$

Inserting this value in (5.19) we obtain

$$e^2 = 96\pi^2 \simeq 947. \tag{6.10}$$

This result is much larger than the experimental value, that is,

$$e_{\mathrm{R}}^2 \simeq \frac{4\pi}{137} \simeq 0{\cdot}092,$$

but this is not very surprising. Our value is unrenormalized and it was calculated by introducing only one fermion field. Moreover, a few simplifying assumptions have been used.

The interesting point is that the calculation gives a number which apparently does not depend crucially on the strength of the fundamental interactions $F_j(\boldsymbol{x})$.

7. Conclusion

The present article can be summarized as follows.

(i) Arguments have been given showing that the photon might be considered as a collective excitation of fermion fields.

(ii) Using considerations of gauge invariance, we proposed to start by writing an action which describes the basic nonlocal gauge invariant interactions of a fermion field (say the electron field) with itself. The interactions depend on unknown short range functions $F_j(x)$.

(iii) Following ideas of Bell and Skyrme, we gave prescriptions for calculating Green functions when the interaction is nonlocal.

(iv) We claimed that by breaking the gauge invariance of this action, it is possible to derive a self-consistent one-body Green function very similar to the Dirac propagator (for instance by linearizing the action: generalized Hartree–Fock approximation).

(v) We found that, in this case, the two-body Green function has a pole. The corresponding composite particle can be considered as a bound state of one fermion and one antifermion; its mass is zero; it may be identified with the photon and we showed that near the pole, the T matrix behaves like a photon propagator.

(vi) A crude estimation of the unrenormalized fine structure constant is given but as could be expected the result is much larger (10 000 times) than the experimental value.

(vii) Thus, the usual electron and photon field theory can be reconstructed; the long range Coulomb forces appear here as resulting from short range self-interactions of the fermion field with itself.

(viii) Since the interactions are nonlocal, we may hope to eliminate, in this way, all divergences from the theory.

However, the theory remains very crude. All fermion fields should be included; the self-consistent symmetry-breaking process involves not only the gauge invariance but also the isospin.

More detailed information must be obtained concerning the basic interactions. The weak interactions may well be a residue of these basic interactions and this possibility also merits exploration.

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Appendix 1

When a Lagrangian exists, the fermion Green functions can be calculated by using a Bell–Skyrme formalism (equations (3.3) and (3.4)). Here is a proof of this property.

Let us write the Lagrangian as a sum of two terms:

$$\mathscr{L}_\psi(x) = \mathscr{L}^0_\psi(x) + \mathscr{L}^{\mathrm{I}}_\psi(x), \tag{A.1}$$

(the index ψ indicates that the field which appears in $\mathscr{L}(x)$ is denoted by $\psi(x)$).

The free fields are solutions of the equation:

$$\frac{\hat{c}\mathscr{L}^0_\psi(x)}{\hat{c}\psi(x)} = 0 \tag{A.2}$$

and the Green function is given by:

$$G(x_1, \ldots, x_n | y_1, \ldots, y_n)$$
$$= (-\mathrm{i})^n N^{-1} \left\langle 0 \left| T\left[\left\{ \exp\left(\mathrm{i} \int \mathscr{L}^{\mathrm{I}}_\psi(x)\, \mathrm{d}^4x \right) \right\} \psi(y_1) \ldots \psi(y_n) \bar{\psi}(x_1) \ldots \bar{\psi}(x_n) \right] \right| 0 \right\rangle$$
$$N = \left\langle 0 \left| T\left\{ \exp\left(\mathrm{i} \int \mathscr{L}^{\mathrm{I}}_\psi(x)\, \mathrm{d}^4x \right) \right\} \right| 0 \right\rangle, \tag{A.3}$$

where the fields $\psi(x)$ and $\bar{\psi}(x)$ are the free fields.

Conversely, the Bell–Skyrme formulae give:

$$G(x_1, \ldots, x_n | y_1, \ldots, y_n)$$
$$= (-\mathrm{i})^n \mathscr{N}^{-1} \left\langle S \left| \left\{ \exp\left(\mathrm{i} \int \{\mathscr{L}_\phi(x) - (\beta + 0)\bar{\phi}(x)\phi(x)\}\, \mathrm{d}^4x \right) \right\} \right.$$
$$\left. \times \phi(y_1) \ldots \phi(y_n) \bar{\phi}(x_1) \ldots \bar{\phi}(x_n) \right| S \right\rangle$$
$$\mathscr{N} = \left\langle S \left| \exp\left(\mathrm{i} \int \{\mathscr{L}_\phi(x) - (\beta + 0)\phi(x)\phi(x)\}\, \mathrm{d}^4x \right) \right| S \right\rangle \tag{A.4}$$

where $\phi(x)$ and $\bar{\phi}(x)$ are anticommuting fields normalized as follows (the fields $a(x)$ and

$\bar{a}(x)$ anticommute with $b(x)$ and $\bar{b}(x)$):

$$\phi(x) = a(x) + \bar{b}(x)$$

$$\phi(x) = \bar{a}(x) - b(x)$$

with

$$[a(x), \bar{a}(y)]_+ = [b(x), \bar{b}(y)]_+ = \beta^{-1}\delta(x-y) \tag{A.5}$$

$$a(x)|S\rangle = 0 \qquad b(x)|S\rangle = 0. \tag{A.6}$$

These expressions can be expanded with respect to $\mathscr{L}^{\mathrm{I}}(n)$. Thus, we see immediately that, if the definitions of $G(x_1, \ldots, x_n | y_1, \ldots, y_n)$ coincide, then $\mathscr{L}^{\mathrm{I}}(n) \neq 0$. Moreover, as it is assumed that $\mathscr{L}^0(x)$ is bilinear with respect to fields, both expressions of the Green function can be calculated by using Wick's theorem.

Thus, to establish the equivalence of the usual theory with the Bell–Skyrme formulation, we have only to compare

$$G^0(x, y) = -\mathrm{i}N^{-1}\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle \tag{A.7}$$

and

$$G^0(x, y) = -\mathrm{i}\mathscr{N}^{-1}\left\langle S \left| \exp\left(\int \{\mathrm{i}\mathscr{L}^0_\phi(x) + (\beta - 0)\bar{\phi}(x)\phi(x)\} \, \mathrm{d}^4x \right) \phi(x)\phi(y) \right| S \right\rangle, \tag{A.8}$$

and to show that these expressions coincide.

The second expression has the form

$$G^0(x, y) = -\mathrm{i}\mathscr{N}^{-1}\left\langle S \left| \exp\left(\mathrm{i}W_0 + (\beta - 0) \int \bar{\phi}(x)\phi(x) \, \mathrm{d}x \right) \phi(x)\bar{\phi}(y) \right| S \right\rangle, \tag{A.9}$$

where

$$W^0 = \int \mathrm{d}^4x \, \mathrm{d}^4y \bar{\phi}(x)\langle x|Z|y\rangle\phi(y) \equiv \langle\psi|Z|\psi\rangle. \tag{A.10}$$

Let ψ_n, $\bar{\psi}_n$ be the eigenvectors of Z, and Z_n the corresponding eigenvalues.

We see immediately that in this representation

$$G^0_{nm} = \mathrm{i}\delta_{nm}\left[\frac{\partial}{\partial\beta'} \ln \langle S| \exp\{(\mathrm{i}Z_n + \beta')\bar{\phi}_n\phi_n\}|S\rangle \right]_{\beta' = \beta - 0} \tag{A.11}$$

where

$$\phi_n = a_n + \bar{b}_n$$

$$\bar{\phi}_n = \bar{a}_n - b_n \tag{A.12}$$

with

$$[a_l, \bar{a}_n] = [b_l, \bar{b}_n] = \beta^{-1}\delta_{l,n}. \tag{A.13}$$

We remark that $(\phi_n)^2 = (\bar{\phi}_n)^2 = 0$.

Thus, since ψ_n and $\bar{\psi}_n$ anticommute:

$$\exp\{\mathrm{i}(Z_n + \beta)\bar{\phi}_n\phi_n\} = 1 + \mathrm{i}(Z_n + \beta)\bar{\phi}_n\phi_n, \tag{A.14}$$

consequently:

$$G^0_{nm} = i\delta_{nm}\left[\frac{\partial}{\partial\beta'}\ln\{1-(iZ_n+\beta')\beta^{-1}\}\right]_{\beta'=\beta-0} = \frac{1}{Z_n+i0}\delta_{nm}. \tag{A.15}$$

Thus we derive from (A.8)

$$G^0 = \frac{1}{Z+i0}. \tag{A.16}$$

In particular, if $\mathscr{L}_\phi(x) = \bar{\phi}(x)(i\gamma\,.\,\partial/\partial x - m)\phi(x)$

$$W_0 = \int \bar{\psi}(p)(\gamma\,.\,p-m)\psi(p)\,d^4p, \tag{A.17}$$

and we find the usual formula

$$G^0(p) = \frac{1}{\gamma\,.\,p-m+i0} \tag{A.18}$$

which is also a direct consequence of (A.7)

$$(p_\nu = i\gamma_\nu).$$

Appendix 2

We want to calculate

$$\lambda_0 A_0 = -\tfrac{1}{96}\,\mathrm{Tr}\int d^4p\,\partial_\nu\{u^\nu(p)(u(p)+i0)^{-1}u_\mu(p)(u(p)+i0)^{-1}u^\mu(p)(u(p)+i0)^{-1}\} \tag{A.19}$$

$$(d^4p = dp_0\,dp_1\,dp_2\,dp_3)$$

$$(\gamma\,.\,p) = (\gamma^\mu p_\mu)$$

when $u(p) = c(\gamma^\mu p_\mu - m)$.

We have

$$\lambda_0 A_0 = -\tfrac{1}{96}\int d^4p\,\partial_\nu[\mathrm{Tr}\{\gamma^\nu(\gamma\,.\,p-m-i0)^{-1}\gamma_\mu(\gamma\,.\,p-m+i0)^{-1}\gamma^\mu(\gamma\,.\,p-m+i0)^{-1}\}] \tag{A.20}$$

We see that the trace in the bracket is a function $p^\nu f(p^2)$ where

$$f(p^2) \simeq \frac{B}{(p^2)^2} \qquad \text{if } p^2 \to \infty, \tag{A.21}$$

on the other hand the poles of $f(p^2)$ are given by

$$p_0 = \pm(m^2+p_1^2+p_2^2+p_3^2-i0)^{1/2}. \tag{A.22}$$

For a given value of p_1, p_2, p_3, we may turn the integration contour in the complex p_0 plane.

We set

$$p_0 = iq_0 \qquad p_j = q_j \qquad (j = 1, 2, 3)$$
$$\gamma^0 = \alpha_0 \qquad \gamma^j = i\alpha_j \qquad (j = 1, 2, 3).$$

Thus

$$[\alpha_l, \alpha_m]_+ = \delta_{mn}.$$

The integral is transformed into

$$\lambda_0 A_0 = -\tfrac{1}{96}i \int d^4q\, \partial_\nu [\mathrm{Tr}\{\alpha_\nu (i\boldsymbol{\alpha} \cdot \boldsymbol{q} - m)^{-1} \alpha_\mu (i\boldsymbol{\alpha} \cdot \boldsymbol{q} - m)^{-1} \alpha_\mu (i\boldsymbol{\alpha} \cdot \boldsymbol{q} - m)^{-1}\}]. \tag{A.23}$$

Now, the matrix is euclidian; integrating the divergence, we find:

$$\lambda_0 A_0 = -\tfrac{1}{96}i \int d^4p\, \partial_\nu (q^\nu f(-q^2)) = -\frac{i\pi^2}{48} B. \tag{A.24}$$

A simple calculation shows that

$$B = -8$$

and therefore that

$$\lambda_0 A_0 = \frac{i\pi^2}{6}. \tag{A.25}$$

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